Given a polynomial F in the j function, Ahlgren, Ono, and Guerzhoy have demonstrated necessary and sufficient conditions for the existence of another polynomial G in $j(z)$ so that $F|U_p$ is congruent to G mod p. We consider similar U_p congruences modulo powers of primes. We also consider other interesting p-adic properties of weight 0 modular functions.